Math Chapter 1 Notes – GM7 - 2018

Ratios

Ratio – A ration is a comparison of two numbers by division
Each number in a ratio is called a term

Example: 6 cups of party mix to 2 cups pretzels
In words: 6 to 2
In symbols: 6:2
As a fraction: 6/2
(NOTE: Do not simplify a ratio by dividing the numerator by the denominator. That would eliminate the ratio!)

Equal ratios (Equivalent Ratios)

Two ratios are equal if you can multiply or divide each term by the same number

2:3 is the same ratio as 4:6 because 2x2 = 4 and 3x2 = 6

<u>To come up with equivalent ratios</u> … simply multiply or divide each term by the same number (other than 0 or 1)

<u>To write a ratio is simplest form</u> … simply divide the two terms by the greatest common factor

Unit Rate

A rate is a ratio that compares two amounts measured in different units.

$\frac{150\ hearbeats}{2\ minutes}$ = 75 beats per minute. *That is the unit rate.*

We use unit rates to calculate miles *per hour, miles per gallon, or prices per pound, dollars per hour, beats per minute, clicks per hour, pounds per square inch, words per minute, RBI's*

Example: $1.20 for 24 oz of ketchup. Divide $1.20 by 24 to find that each ounce is $.05

Here's an easy way to remember it:

$\frac{Thing\ 1}{Thing\ 2}$ <--- Dividing line means *per*

Jack ran 6 miles in 2 hours. Find out his rate per hour.

$\frac{Miles}{Hours}\ \frac{6\ miles}{2\ hours}$ ---> Then simplify 3 miles / 1 hour. 3 miles per hour.

Proportion

A proportion is an equation stating that two ratios are equal

Two ratios form a proportion if they are equivalent.

Example: 7/35 and 21/105 are proportional because 7 x 3 = 21 and 35 x 3 = 105

Cross Products

You can use cross-multiplying to see if two ratios form a proportion.

Example: $\frac{3}{18}$ and $\frac{1}{6}$ Since 3 x 6 = 18 and 1 x 18 = 18, these two fractions are proportional.

To solve a proportion using a variable …

Use cross-multiplying.

$$\frac{10}{3} = \frac{c}{12}$$

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10 x 12 = 120. c x 3 = 3c

120 ÷ 3 = 40, so c = 40

(Note: You can sometimes find the proportion without cross-multiplying … 3 x ? = 12. However, cross-multiplying works in every situation)

Complex Fractions

Examples: $\frac{\frac{1}{3}}{4}$ $\frac{3}{\frac{4}{5}}$ $\frac{\frac{1}{2}}{\frac{3}{5}}$

Fractions with a numerator, denominator, or both that are also fractions. Complex fractions are simplified when both the numerator and denominator are integers

Rewrite each complex fraction as a division problem, then solve to simplify.

$$\frac{\frac{1}{3}}{4} = \frac{1}{3} \div 4 = \frac{1}{3} \div \frac{4}{1} = \frac{1}{3} \times \frac{1}{4} = \mathbf{\frac{1}{12}}$$

If one of the numerators or denominators is a mixed number, write the mixed number as an improper fraction.

$$\frac{1\frac{1}{3}}{\frac{1}{4}} = 1\frac{1}{3} \div \frac{1}{4} = \frac{4}{3} \div \frac{1}{4} = \frac{4}{3} \times \frac{4}{1} = \frac{16}{3} = \mathbf{5\frac{1}{3}}$$

Unit Ratios

A unit ratio is one in which the denominator is 1 unit

$$\frac{\mathbf{12\ \mathit{inches}}}{\mathbf{1\ \mathit{foot}}} \qquad \frac{\mathbf{16\ \mathit{ounces}}}{\mathbf{1\ \mathit{pound}}} \quad \frac{\mathbf{100\ \mathit{centimeters}}}{\mathbf{1\ \mathit{meter}}}$$

You can convert one rate to an equivalent rate by multiplying by a unit ratio or its reciprocal. When you convert rates, you include the units in your computation.

Dimensional analysis -the process of including units of measure as factors when you compute

Example: Convert 12 feet per second into inches per second

$$\frac{\mathbf{12\ \cancel{\mathit{ft}}}}{\mathbf{1\ \mathit{s}}} * \frac{\mathbf{12\ \mathit{inches}}}{\mathbf{1\ \cancel{\mathit{ft}}}}$$

Cross-reduce the like units

$$= \frac{\mathbf{144\ \mathit{inches}}}{\mathbf{1\ \mathit{s}}}$$

Proportional Quantities – Two quantities are proportional if they have a constant ratio or unit rate. Example:

Typically, when you buy things at a grocery store, you get charged the same amount for each item. This is a proportional quantity. If laundry soap costs $5 per bottle, then 4 bottles would be $20.

Nonproportional – Two quantities in which the rate is not constant.

For example: 16 oz of cereal costs $2, but 32 ounces costs $3.50, 48 ounces costs $4.75. (The more you buy, the more you save).

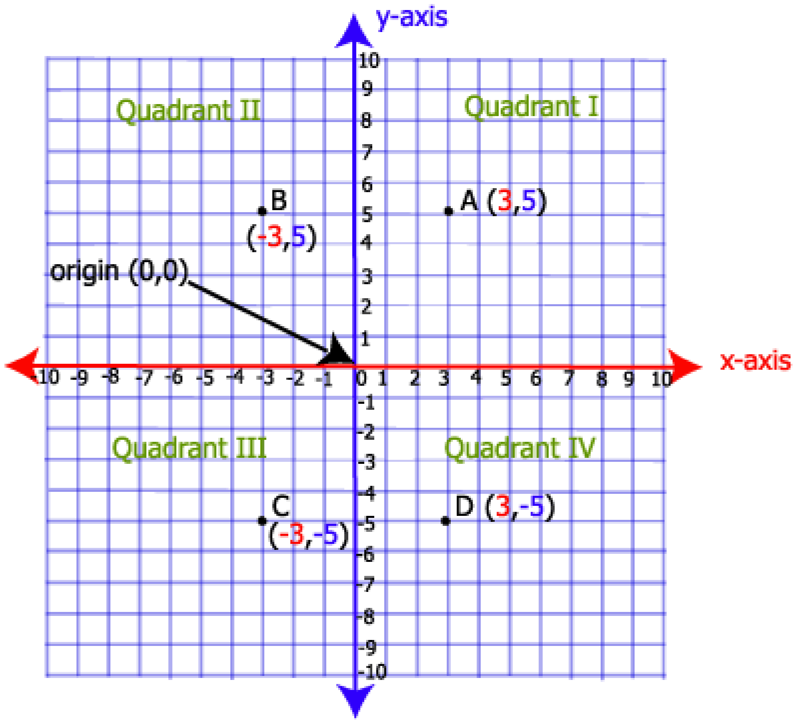
If the ratios (fractions) can be simplified to the same value, then they are proportional

5/1 10/2 20/5 etc … can all be simplified to 5

If, when the ratios are simplified, they do not equal the same value, then they are nonproportional

5/2 11/3 15/4 etc.

Graphing



Independent and Dependent Variables
Independent (*x*) what is given and doesn't change (the input variable)
Dependent (*y*) Depends on *x* and changes (increases or decreases) depending on *x* (the output or the answer)

Think of an input-output table

The input is the independent variable and the output is the dependent variable. The output is dependent on the input.

Input	Output
1	
2	
3	
4	

Rule: Input x $4

Example:
Cali makes $8 an hour babysitting.

What is the dependent variable? (Amount she makes)

What is the independent variable? (Number of hours she works)

Linear function – A function whose graph is a line.

Graphing Functions: Making a graph from an equation.

Note: If the equation is not in the format **y = mx + b**, put it in that format using inverse operations.

Example: y = 2x + 1

1) Make a t-chart

x	y

2) Make up numbers for x and plug them into the equation to find the y value.
NOTE: You can use any values you want! However, I like to keep them small to make the numbers more manageable.

y = 2x + 1

x	y
1	
0	
-1	
2	

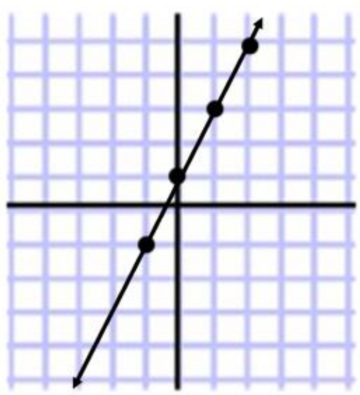
y = 2(1) + 1
y = 2 + 1
y = 3

x	y
1	**3**
0	
-1	
2	

Complete the table by plugging in values of x into the equation.

x	y
1	3
0	1
-1	-1
2	5

3) Plot the points on the coordinate grid and connect the points to form a line.



Relationships that have straight-line graphs are called **linear relationships**. If an equation can be graphed as a straight line, it is a **linear equation**.

A linear relationship has a **constant rate of change**. This means that the rate of change between two points is the same, or constant. When two variable quantities have a constant ratio, their relationship is called a **direct variation**.
Constant Ratio = Constant of variation = constant of proportionality (all the same thing)

Example 1:

5	10	15	20	25	30
1	2	3	4	5	6

Two quantities have a **proportional relationship** if they have a constant ratio and constant rate of change.
In the table above, the ratio is 5:1, and it is constant throughout.

To determine if two quantities are proportional, compare the ratio for several points to determine if there is a constant ratio.

Example 2:

5	**10**	**15**	**20**	**25**	**30**
1	**2.5**	**3.6**	**4.9**	**5.2**	**5.9**

In this table, the quantities are **not proportional.** There is no constant ratio or constant rate of change.

Typically, proportional relationships can be found in two ways.

Arithmetic Sequence: a sequence where the **same number** is added or subtracted from the previous number or term.
Example:

1	**2**	**3**	**4**	**5**	**6**
3	**4**	**5**	**6**	**7**	**8**

Geometric Sequence: a sequence of numbers where each term after the first is found by multiplying or dividing the previous one by the same amount. (Example: Table 1 above)

However, there are more complex linear equations where **both** types of sequences are used.

SLOPE
Slope is a rate of change.
It can be positive (slanting upward) or negative (slanting downhill)

It is found by finding the ratio of the rise versus the run

Slope = $\frac{rise}{run} = \frac{up}{over}$

If a line moves up four points for every 3 points it moves to the right, then the slope of the line would be $\frac{4}{3}$

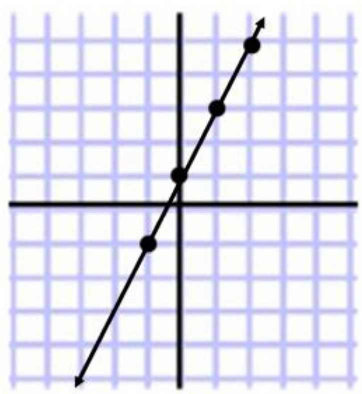
Slope Formula

Slope is represented by the variable *m*.

The slope (m) of a line passing through points (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x coordinates.

Formula: $\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$

Example:



1) Find two points on the line. (It doesn't matter which two points, as long as they are found at intersections).

How about (0, 1) and (1, 3)

x_1 is 0 and x_2 is 3

y_1 is 1 and y_2 is 3

Plug these into the formula to find the slope.

$\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$ $\boldsymbol{m} = \frac{\mathbf{3 - 1}}{\mathbf{1 - 0}}$ $\boldsymbol{m} = \frac{\mathbf{2}}{\mathbf{1}}$

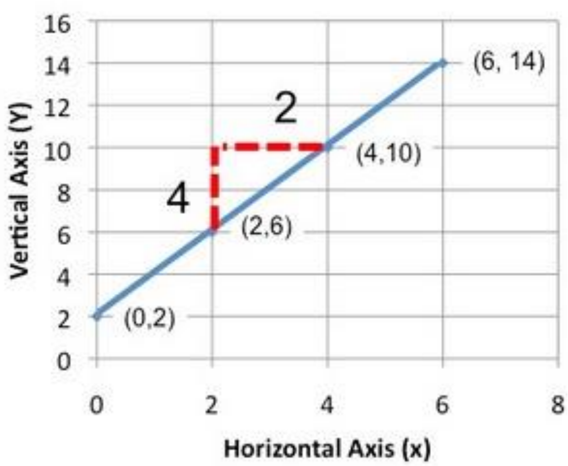
The slope is 2/1. For every two the line moves up, it moves over to the right once.

It doesn't matter which point you define as (x_1, y_1) and (x_2, y_2). However, the coordinates of both points must be used in the same order.

Note: The Slope Formula is useful when you only have the coordinates and you don't want to graph it. It is generally easier to find slope from a graph than to plug the coordinates into the formula.

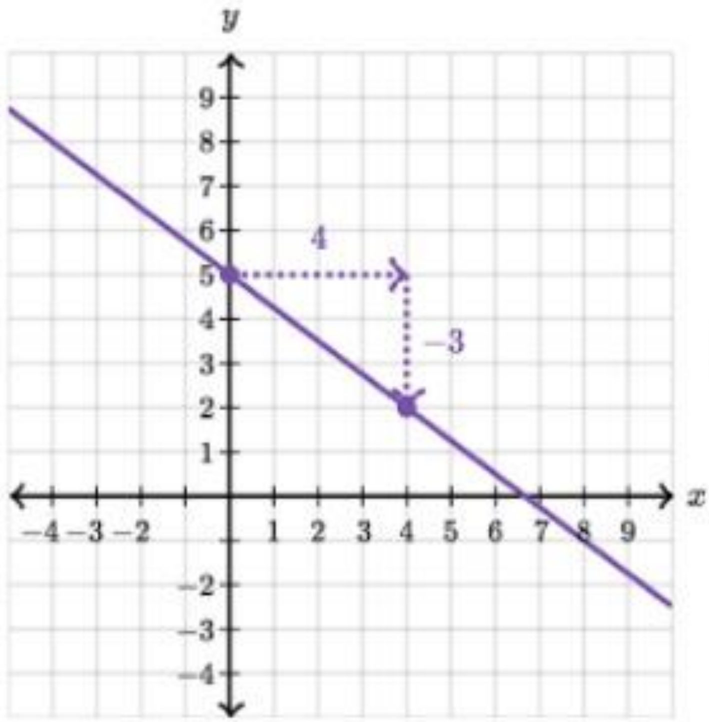
IMPORTANT NOTE: If finding slope from a graph, please be aware of the value of the intervals. In the graph above, each interval (square) is worth 1. However, in some graphs, the intervals could be 5, 10, 100 or anything.

Example:



In this example, the slope is actually 4/2 or 2/1. Although the graph goes up two boxes for every two it goes over, the value of the intervals is different.

Example 2:



(x_1, y_1) and (x_2, y_2)

(0, 5) and (4, 2)

$\boldsymbol{m} = \frac{y_2 - y_1}{x_2 - x_1}$ $\boldsymbol{m} = \frac{\mathbf{2-5}}{\mathbf{4-0}}$ $\boldsymbol{m} = \frac{\mathbf{-3}}{\mathbf{4}}$

Note how the slope is **negative**, which corresponds to the downward slope of the line.